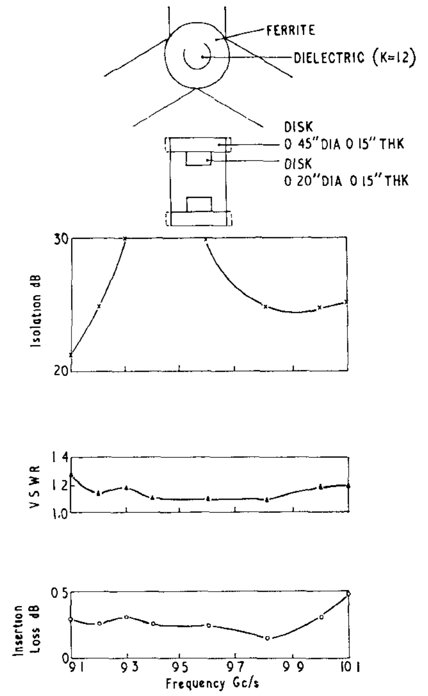

Fig. 6—Circulation obtained with ferrite disks and dielectric.

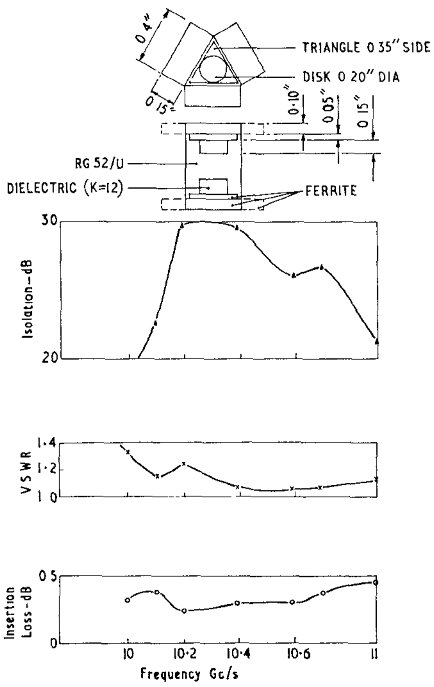

Fig 7—Circulation obtained with the triangular ferrite configuration and dielectric.

ever, where the thickness is greater than 0.10 inch the diameter must rapidly be reduced to approach 0.20 inch at a distance of 0.15 inch–0.20 inch from the narrow walls in order to prevent complex behavior.

Similar results were obtained with triangular pieces of ferrite instead of disks. With triangles, 0.4 inch side, 0.10 inch thick, a broad-band mode of circulation occurred in the frequency range 8–10 Gc. Increasing the thickness to 0.15 inch deteriorated that mode, shifted the frequency range down, and introduced a mode of opposite circulation at the higher frequencies. But the performance of the original mode was improved without introducing the higher one by using smaller triangles on top of the 0.4 inch side, 0.10 inch thick pieces. Thus, the behavior is similar to that of the disks, a convergent tapering of the ferrite towards the center of the junction being necessary to maintain a single broadband mode of circulation.

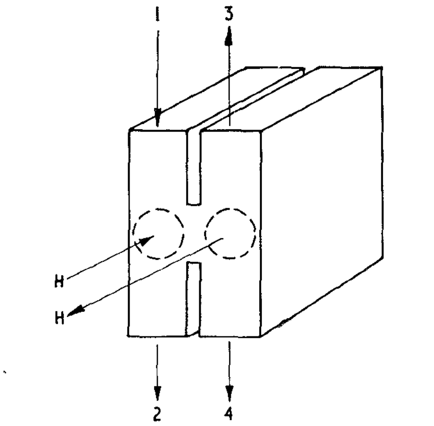

Fig. 8—Two oppositely polarized folded E-plane circulators forming a very compact 4-port circulator.

Two matching techniques were used to improve the performance of these devices. Adding dielectric on top of the ferrite, and placing small rectangular ferrite slabs against the sides of the triangular pieces are methods which can be used separately or together. Examples of both these configurations giving bandwidths of about 10 per cent are shown in Figs. 6 and 7. The addition of dielectric on top of the ferrite is the preferable method, since this will permit greater compactness, and a further reduction in volume may be possible if a reduced-height waveguide junction is used.

It has been established that a single mode of circulation does exist over the waveguide pass band using an E-plane 3-port junction with thin ferrite disks placed against the narrow walls. Compact circulators of this type may offer a realistic alternative to the stripline 3-port circulator. This is particularly relevant when a "straight-through" geometry is sought, as would be required in a dewar application. For this purpose two folded E-plane 3-port devices with opposite senses of polarization would form a very compact 4-port circulator, as shown in Fig. 8.

Finally, it is interesting to speculate on the fundamental difference between the E- and H-plane 3-port circulators. They depend respectively on the longitudinal and transverse components of the RF magnetic field which have opposite frequency dependence. This would suggest that the E-plane device would have a better performance at the lower end of the waveguide pass band, and the H-plane a better performance at the upper end.

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Comments on "Pulse Waveform Degradation Due to Dispersion in Waveguide"*

Recent work at The Hallicrafters Company concerned with the analysis of the transmission of pulsed electromagnetic energy through dispersive media has caused the writers to review the above work of R. S. Elliott.[1] In this review it was noted that (14) of that work contains an error (which has been brought to the attention of R. S. Elliott, who agrees that it does exist). This equation should read

$$F(t) = \frac{1}{\sqrt{2}} \sqrt{X^2 + Y^2}, \tag{1}$$

where

$$X = C(A_1) - C(A_2) \tag{2}$$

$$Y = S(A_2) - S(A_1) \tag{3}$$

with

$$C(A) = \int_0^A \cos\left(\frac{\pi}{2} y^2\right) dy = \text{Cosine Fresnel Integral} \tag{4}$$

$$S(A) = \int_0^A \sin\left(\frac{\pi}{2} y^2\right) dy = \text{Sine Fresnel Integral} \tag{5}$$

$$A_1 = \frac{x+1}{a\sqrt{\pi/2}} \tag{6}$$

$$A_2 = \frac{x-1}{a\sqrt{\pi/2}} \tag{7}$$

$$x = \frac{2t'}{T} \tag{8}$$

$$a = \frac{4}{T}\sqrt{BL} \tag{9}$$

$$t' = t - AL. \tag{10}$$

Eq. (1) has also been obtained independently and at about the same time by R. O. Brooks of the Raytheon Company.

The above is for an input pulsed carrier turned on at time $= -T/2$ and of duration T. For the same pulsed carrier, but turned on at time $t=0$, the result is given by (1) with X and Y given by

$$X = C(A_1') - C(A_2') \tag{11}$$

$$Y = S(A_2') - S(A_1'), \tag{12}$$

where

$$A_1' = \frac{x}{a\sqrt{\pi/2}} \tag{13}$$

$$A_2' = \frac{x-2}{a\sqrt{\pi/2}}. \tag{14}$$

Numerical computations of (1) using (11) and (12) for the cases of $a=0$, 0.032, 0.10, 0.32, 0.50, and 1.00 are shown in Fig. 1, and reveal that these shapes are practically identical to those of Elliott except for the large values of a. The computations were performed using the Fresnel Integral Tables of Pearcy,[2] and are tabulated in Table I.

* Received May 6, 1963.

[1] R. S Elliott, "Pulse waveform degradation due to dispersion in waveguides," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-5, pp. 254–257; October, 1957.

[2] T. Pearcy, "Table of the Fresnel Integral," Cambridge at the University Press, Cambridge, England, 1956.

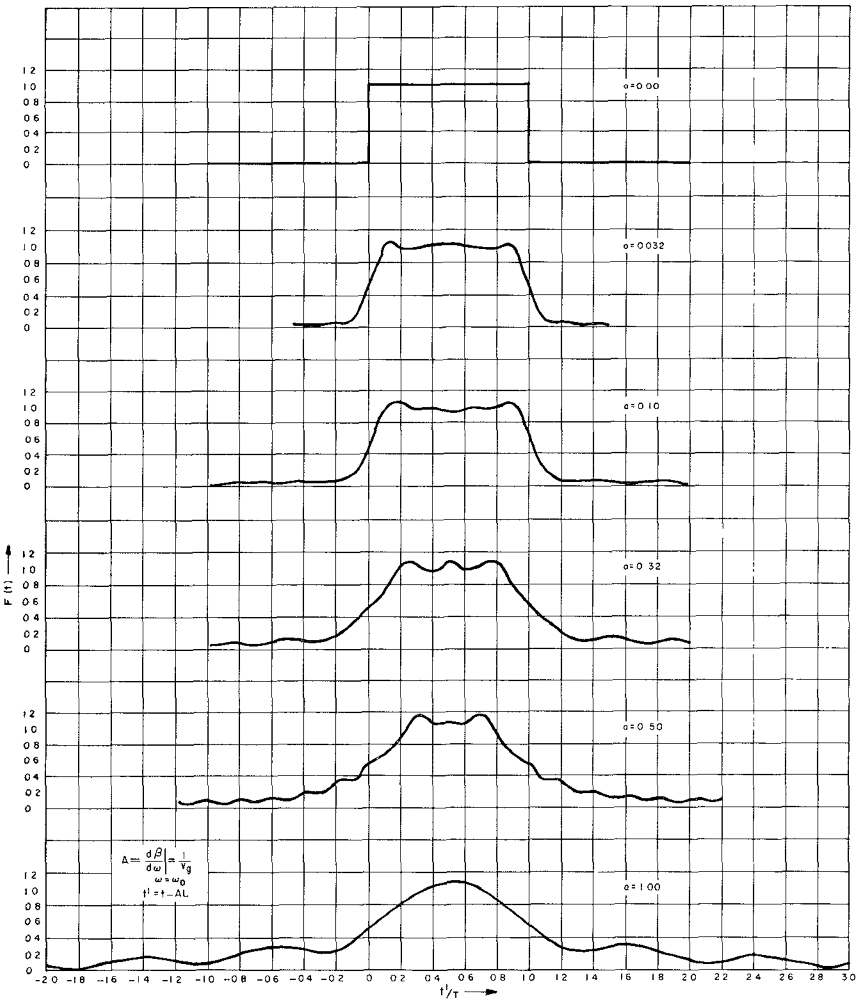


Fig. 1—Degraded waveforms.

TABLE I

TABULATION OF $\sqrt{\frac{X^2+Y^2}{2}}$

t'/T	$a=0.00$	$a=0.032$	$a=0.10$	$a=0.32$	$a=0\ 50$	$a=1.00$
0.0	1.00	0.495	0.495	0.523	0.538	0 523
0.1	1.00	1.022	1.020	0.791	0 672	0.691
0.2	1.00	0.973	1.024	1.080	0.887	0.846
0.3	1.00	0.984	0.981	1.044	1 180	0.971
0.4	1.00	1.000	0.992	0.967	1.051	1.051
0.5	1.00	1.016	0.946	1 098	1 045	1.080
0.6	1.00	1.000	0.992	0.967	1.051	1 051
0.7	1.00	0.984	0.981	1.044	1 180	0 971
0 8	1.00	0.973	1 024	1.080	0.887	0 846
0.9	1.00	1.022	1.020	0.741	0.672	0.691
1.0	1 00	0.495	0 495	0.523	0.538	0 523
1.1	0.00	0.045	0.149	0.350	0 326	0.366
1.2	0.00	0 026	0 065	0.209	0 318	0.254
1.3		0.012	0.040	0.111	0.170	0.224
1.4		0 014	0 043	0 107	0 208	0.253
1 5		0.010	0.035	0.118	0 105	0.285
1.6			0.018	0 089	0 152	0.291
1.7			0 017	0 042	0.075	0.266
1.8			0 025	0.050	0 119	0.214
1.9			0 019	0.072	0 061	0.147
2.0			0 007	0.060	0.097	0 082
2.1					0 052	0.064
2.2					0 091	0 103
2.4						0.139
2.4						0.155
2.5						0 148
2.6						0.122
2.7						0 081
2.8						0 038
2 9						0.034
3.0						0 067

Additional Points

t'/T	$a=0\ 50$
0.25	1.095
0.35	1.135
0.45	1.035
0 55	1.035
0.65	1.135
0.75	1 095
1.05	0.375
1.15	0.360
1.25	0.206

The shapes are symmetrical with respect to $t'/T=0.50$.

However, examination of these shapes reveals that an output exists for all values of time. This observation, an output occurring prior to an input, is disquieting since it violates the law of causality.

The reason for this is that the use of the approximation of (7) of Elliott's paper for the phase constant $\beta(\omega)$ gives rise to an effective transfer function, $H(\omega)$, of the waveguide approximated by

$$H(\omega) \approx \exp\left\{-j[\beta_0 + A(\omega-\omega_0) - B(\omega-\omega_0)^2]L\right\}. \quad (15)$$

This transfer function is not physically realizable. This can be seen very readily by determining the output response due to an input impulse (delta) function of a hypothetical black box characterized by this transfer function.

This response, designated by $h(t)$, is given by the Fourier transform of the transfer function[3]

$$h(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(\omega)e^{j\omega t}d\omega. \quad (16)$$

Insertion of (15) into (16), reduction into trigonometric functions, and use of Dwight's[4] integrals 858.564 and 858.565 gives

$$h(t) = \frac{1}{2\pi}\sqrt{\frac{\pi}{BL}}\exp\left\{j\left[\omega_0 t - \beta_0 L + \frac{\pi}{4} - \frac{(t-AL)^2}{4BL}\right]\right\} \quad -\infty \le t \le \infty. \quad (17)$$

Inspection of (17) shows that an output exists prior to an input, which is physically impossible. Hence the transfer function $H(\omega)$ given by (15) is physically nonrealizable. Use of Bode's[5] physical realizable criteria to the approximate transfer function of (15) also shows it to be physically nonrealizable.

It is interesting to note that the convolution integral[3] can be used to determine the output $f(t)$ via integration in the time domain, *i.e.*,

$$f(t) = \int_{-\infty}^{\infty} e(t-\tau)h(\tau)d\tau, \quad (18)$$

where $e(t-\tau)=$ input to waveguide at time $t-\tau$, $h(\tau)=$response of waveguide to impulse function [given by (16)]. Eq. (18) gives the same result for $F(t)$ as given by (1) without the necessity of using the Foster and Campbell[6] pair 731.1.

Since (1) violates the law of causality its adequacy to describe the actual output of a waveguide must be used with this violation in mind. This situation is somewhat similar to the case of the idealized low-pass filter[7] which predicts an output prior to an input.

[3] J. C. Hancock, "An Introduction to the Principles of Communication Theory," McGraw-Hill Book Co , Inc., New York, N. Y., pp. 125–126; 1961.

[4] H. B. Dwight, "Tables of Integrals and Other Mathematical Data," The Macmillan Co , New York, N. Y., 4th ed., p. 220; 1962

[5] H. W. Bode, "Network Analysis and Feedback Amplifier Design," D. Van Nostrand Co., Inc , New York, N. Y., p. 106; 1945.

[6] G. A. Campbell and R. M. Foster, "Fourier Integrals for Practical Applications," D. Van Nostrand, Inc , New York, N. Y., 1948.

[7] E A. Guillemin, "Communication Networks," John Wiley and Sons, Inc., New York, N. Y., vol. 2, pp. 477–486, 1935.

The accuracy with which (1) describes the actual output can be determined by comparing it with the solution obtained by using the exact transfer function of the waveguide. It is believed that this solution has not as yet been obtained in a closed form. However, some work for the case of a step function carrier utilizing numerical integration of the exact transfer function of the waveguide has been performed for a few specific cases.[8]

It is interesting to compare the waveforms predicted from this work with those utilizing the approximate transfer function given by (15). For a step function input of

$$e(t) = E_0 \sin \omega_0 t \, 1(t), \qquad (19)$$

where $1(t)$ is the step function defined by

$$1(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0, \end{cases} \qquad (20)$$

the output of a waveguide characterized by the approximate transfer function of (15), via (18) or following Elliott, is

$$f(t) = \frac{E_0}{2} \exp \{ j[\omega_0 t - \beta_0 L] \} \operatorname{erfc} Z_0, \quad (21)$$

where it is understood that the imaginary part of (21) is to be taken for $f(t)$, and with

$$Z_0 = -\frac{(t - AL)(1 + j)}{2\sqrt{2BL}}, \qquad (22)$$

where

$$\operatorname{erfc} Z_0 = \frac{2}{\sqrt{\pi}} \int_{Z_0}^{\infty} e^{-z^2} dz. \qquad (23)$$

Integration of (23) gives for the output envelope $F(t)$

$$F(t) = |f(t)| = \frac{E_0}{2} \cdot \sqrt{1 + 2[C^2(A_1') + S^2(A_1') + C(A_1') + S(A_1')]}. \quad (24)$$

where A_1' is given by (13) and $C(A)$ and $S(A)$ by (4) and (5), respectively.

Plots of $F(t)$ via (24) (which is essentially based on Elliott's work), and of $F(t)$ (based on the work and Figs. 3 and 4 of Cohn[8]) are shown in Fig. 2. These plots are for the two cases

$$\frac{\omega_0}{\omega_c} = 1.10, \qquad \frac{L}{\lambda_{v0}} = 0.875$$

and

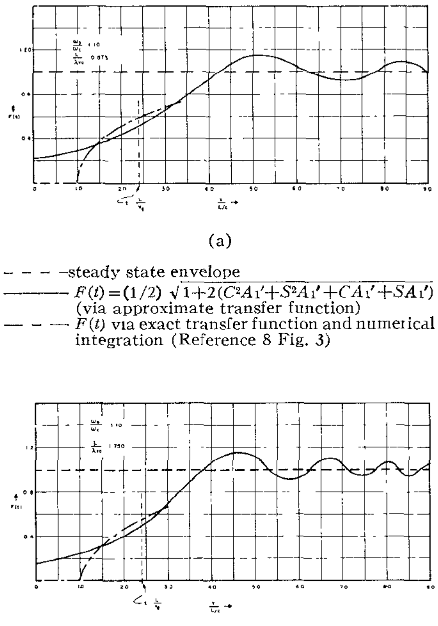
$$\frac{\omega_0}{\omega_c} = 1.10, \qquad \frac{L}{\lambda_{v0}} = 1.750,$$

where λ_{v0} = vacuum wavelength of excitation $= c/f_0$, $f_0 = \omega_0/2\pi$, c = speed of light.

The work of Cohn indicates that the output pulse starts at the time L/c, not L/v_g, where

$$v_g = \text{group velocity} = \frac{1}{A} = \left(\frac{d\beta}{d\omega}\bigg|_{\omega=\omega_0} \right)^{-1} = \frac{c\left[\left(\frac{\omega_0}{\omega_c}\right)^2 - 1\right]^{1/2}}{\left(\frac{\omega_0}{\omega_c}\right)}.$$

[8] G. I. Cohn, "Electromagnetic transients in Waveguides," *Proc. NEC*, Chicago, Ill., September 29–October 1, 1952, vol. 8, pp. 284–295.



(a)

– – – steady state envelope
——— $F(t) = (1/2)\sqrt{1+2(C^2A_1'+S^2A_1'+CA_1'+SA_1')}$ (via approximate transfer function)
— — — $F(t)$ via exact transfer function and numerical integration (Reference 8 Fig. 3)

(b)

– – – – steady state envelope
——— $F(t) = (1/2)\sqrt{1+2(C^2A_1'+S^2A_1'+CA_1'+SA_1')}$ (via approximate transfer function)
– - – - – $F(t)$ via exact transfer function and numerical integration (Reference 8 Fig. 4)

Fig 2—Output waveform of a waveguide due to a step function carrier input.

For the case of Fig. 2, $c/v_g = 2.40$. This is in accordance with the meaning of wave front velocity.[9] The plots of Fig. 2 indicate that the waveform envelope predicted using the approximate transfer function of the waveguide given by (15) approximates that obtained via numerical integration, using the exact transfer function of the waveguide quite well for the time range indicated. The envelope for $t < 0$ via the approximate transfer function is not shown but is nonzero. The close agreement for this step function input for the specific cases of ω_0/ω_c and L/λ_{v0} indicate that the use of the approximate transfer functions gives a good approximation for the output waveform for the time ranges available for comparison. Generalizing from this comparison, one would expect the same approximation to be as good for the pulsed carrier input and hence that for $t > L/c$ that the envelope shapes of Fig. 1 are good approximations to the output pulse shapes of a waveguide. It would be in order, however, to confirm this generalization by obtaining an exact closed form solution for the pulsed case.

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[9] J. A Stratton, "Electromagnetic Theory," McGraw-Hill Book Co., Inc., New York, N. Y., p. 337; 1941.

Temperature Stabilization of Gyromagnetic Couplers*

A gyromagnetic coupler using a single-crystal YIG sphere as a coupling element suffers from two significant sources of temperature instability. One of these is anisotropy drift,[1] a characteristic that is internal to the coupling element, since it stems directly from temperature induced variations in crystalline anisotropy.[2] The other is appropriately characterized as external; it derives from temperature induced variations in the magnetic biasing source. Either or both of these variations will result in a change in the resonant frequency of a gyromagnetic coupler. The 3-db bandwidth of a low loss YIG coupler may be of the order of 40 Mc, hence a change in resonant frequency of as little as 5 Mc will be detected as an increase in insertion loss at the original frequency. It is therefore quite desirable that the variations which contribute to this instability be reduced to a minimum. Means have been developed for eliminating both of these instabilities, thus rendering the gyromagnetic coupler a much more practical device under a variety of environmental conditions.

For power limiting applications at relatively high power levels, such as those previously reported by the authors at *C*-band frequencies,[3,4] a problem arises through high power heating of the YIG crystal. Under some conditions the resultant anisotropy drift can have an appreciable effect on the operation of the device. The authors observed this effect in a test arrangement in which a high power pulse of variable amplitude is closely followed by a low power pulse ($\approx$1 mw) of constant amplitude. With the YIG crystal (23 mils in diameter) randomly oriented and a constant dc magnetic field applied, a drift in the frequency of optimum transmission for the low power signal of more than 40 Mc has been observed when the high power signal is raised from 0 to 1000-w peak, 1-w average.

Crystalline orientation is the direct solution to the problem of anisotropy drift. With the biasing field restricted to a 110 plane, profitable use can be made of the equation[5]

$$H_{\text{eff}}^2 = (H_0 + AH_a)(H_0 + BH_a)$$

where second-order anisotropy effects are neglected and spherical geometry is assumed.

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[1] The problem of anisotropy drift in lithium ferrite single crystals in gyromagnetic couplers was discussed by S. Okwit at the 1962 PGMTT National Symposium.

[2] A. M Bozorth, "Ferromagnetism," D. Van Nostrand Company, Inc , Princeton, N. J.; 1951.

[3] J. Clark and J. Brown, "The gyromagnetic coupling limiter at *C*-band," IRE Trans. on Microwave Theory and Techniques (*Correspondence*), vol. MTT-10, pp 84–85, January, 1962.

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[5] P. J. B Clarricoats, "Microwave Ferrites," John Wiley and Sons, Inc., New York, N. Y., 1961.